13TH LATVIAN MATHEMATICAL CONFERENCE

January 24–25, 2025
UL House of Science, Jelgavas street 3, Riga

13. LATVIJAS MATEMĀTIKAS KONFERENCE

24.–25. janvāris, 2025
LU Zinātņu māja, Jelgavas iela 3, Rīga

ABSTRACTS
TĒZES

UNIVERSITY OF LATVIA

Latvian Mathematical Society
Faculty of Science and Technology, University of Latvia
Institute of Mathematics and Computer Science, University of Latvia

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ABSTRACTS

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TĒZES

University of Latvia
Riga 2025

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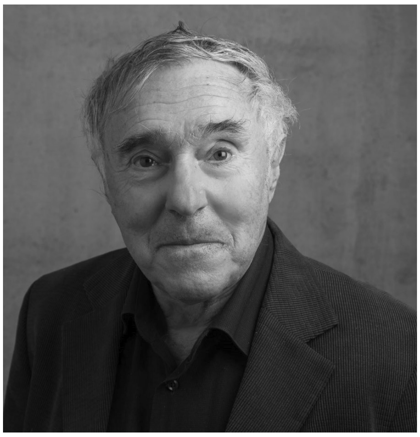
https://doi.org/10.22364/lmc.13.a25
ISBN 978-9934-36-341-2
ISBN 978-9934-36-342-9 (PDF)

IN MEMORIAM:
ALERKSANDRS ŠOSTAKS (1948–2024)



Aleksandrs Šostaks was born in 1948 in Riga. He obtained a higher education in mathematics from the State University of Latvia in 1971. In 1974, he graduated from the doctoral program of the State University of Latvia and in 1975 earned the title of Candidate of Physical and Mathematical Sciences by defending his dissertation at the Peoples' Friendship University in Moscow. In 1992, Aleksandrs Šostaks obtained the degree of habilitated doctor of mathematics at the University of Latvia, and in 1993, he was awarded the academic title of professor.

Aleksandrs Šostaks devoted significant part of his career to the University of Latvia. He began working at the Faculty of Physics and Mathematics in 1971 as an assistant, later becoming a lecturer, assistant professor, associated professor and professor. He actively engaged in faculty life, leading the Department of Mathematical Analysis and the professional study program "Mathematician Statistician". During several years he was an active member of the faculty council representing the Department of Mathematics.

His research activities were associated with two scientific institutes – from 1991 to 2006 with the Institute of Mathematics of the Latvian Academy of Sciences and the University of Latvia, and since 2006 until the last day of his life with the Institute of Mathematics and Computer Science of the University of Latvia, where he served as a lead researcher.

Research topics of Aleksandrs Šostaks included topology, category theory, fuzzy logic, multivalued mathematical structures, and their applications. He is the author of around 200 scientific publications. Under his supervision, 10 doctoral theses were developed and successfully defended (6 in Latvia, 1 in Spain, 1 in Germany, 1 in Moscow, and 1 in Kyrgyzstan). In addition, two of his students have almost completed their dissertations. Last year, under professor Šostaks' guidance, two new doctoral students began their research work very successfully, receiving awards for scientific publications at international conferences in their first year of study.

Aleksandrs Šostaks' research primarily focuses on fuzzy topology, with several notable contributions over the years. In the 1990s he reviewed two decades of developments in fuzzy topology, outlining foundational concepts and key results, studied basic structures in fuzzy topology and introduced fuzzy syntopogeneous structures, which generalize topological spaces. In 2000s, he explored multivalued topologies using fuzzy logic, enhancing the understanding of complex set-theoretic interactions and developed M-approximative operators, focusing on approximate reasoning in fuzzy systems. In next decade Šostaks analyzed the historical development and trends in fuzzy set theory, providing insights into its evolution and future. He investigated rough set theory's integration with fuzzy sets, emphasizing topological aspects. Šostaks revisited fuzzy metric spaces, refining models and contributing to the understanding of pseudometric and metric structures in fuzzy contexts.

Aleksandrs Šostaks has contributed significantly to the field of fuzzy mathematics through numerous collaborations. During the 1990s, he partnered with Mark Burgin on fuzzification of continuous functions, and with Yu.H. Bregman and B.E. Shapirovskiĭ on decomposing topological spaces and worked with Tomaz Kubiak on lower set-valued fuzzy topologies, extending traditional topology into the fuzzy domain. He also collaborated with Juris Steprāns on compactness in topological spaces under product operations. In the late 1990s and early 2000s, his research with Svetlana Asmuss focused on fuzzy approximation theory.

In the mid-2000s, Šostaks and Martin Kalina explored differentiability within fuzzy logic, advancing theoretical foundations for applying fuzzy logic to differential calculus. His work with Radko Mesiar and Martin Štěpnička in 2012-2013 highlighted advancements in fuzzy set theory. In 2014, he partnered with Sang-Eon Han and In Soo Kim on approximate-type systems using L-relations, and with Irina Perfilieva on fuzzy functions and lattice-valued theories. His collaboration with Banu Pazar Varol and Halis Aygün in 2011-2012 integrated soft set theory with topology.

From 2011 to 2017, he worked with Mati Abel and Ingrīda Uļjane on bornological structures in fuzzy set theory, and with Patrik Eklund, Ingrīda Uļjane, and Aleksandrs Eļkins on fuzzy relational mathematical morphology. His research with Juan-José Miñana and Oscar Valero in 2016-2020 advanced fuzzy metric theory. In 2021-2023, he collaborated with Māris Krastiņš on using fuzzy concept lattices in risk analysis. Recently, his work with Olga Grigorenko and Raivis Bēts focused on parameterized metrics and their applications. Lastly, his research with Ksenija Varfolomejeva explored fuzzy morphological operators in L-fuzzy groups. Through these collaborations, Šostaks has significantly enhanced the understanding and application of fuzzy logic in various mathematical and practical contexts.

Professor Šostaks has developed 12 teaching materials and taught more than 20 lecture courses. He taught not only at the University of Latvia but, at various times, also at Liepaja University, the RISEBA university, and the Transport and Telecommunications Institute in Latvia, as well as abroad – in South Africa (at Rhodes University in 1993) and Canada (at York University in 2006). In addition to his teaching responsibilities at York University, he worked at the Fields Institute in 2006, conducting research there. From 1978 to 1979, he worked as a research scientist at the University of Zagreb in Croatia.

Professor Aleksandrs Šostaks was a recognized scientist in Latvia and worldwide, with a wide network of collaborators in the Czech Republic, Slovakia, Spain, Canada, Germany, Estonia, China, South Korea, and Turkey. He was a member of the editorial boards of several international scientific journals and a permanent member of the program committees of many international conferences (EUSFLAT, IFSA, ESCIM, FSTA, etc.). Under his guidance, many fundamental research projects funded by the Latvian Science Council and EU structural funds were successfully implemented. He was one of the founders of the Latvian Mathematical Society in 1993, a board member since its inception, and served as chairman from 1996 to 2001. In 2004, Aleksandrs Šostaks was elected as a corresponding member of the Latvian Academy of Sciences, and in 2017, he became a full member. In 2017, he was awarded the title of State Emeritus Scientist and received the Piers Bohl Award from the Latvian Academy of Sciences. He is the recipient of the 2018 award from the University of Latvia for establishing a scientific school.

Aleksandrs Šostaks' legacy is marked by his profound impact on the field of mathematics, his dedication to mentoring the next generation of scholars, and his enduring contributions to scientific research and international collaboration. His work has significantly enriched the understanding and application of fuzzy logic and mathematics, leaving a lasting influence on the academic community.

Acta Societatis Mathematicae Latviensis
Abstracts of the 13th Latvian Mathematical Conference, January 24-25, 2025, Riga, Latvia

STRUCTURAL EQUATION MODELING (SEM): THEORY, APPLICATIONS, AND RESEARCH DIRECTIONS

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Structural Equation Modeling (SEM) is a versatile statistical framework designed to test hypotheses regarding relationships among latent variables, which are typically represented through observed, or manifest, variables. SEM encompasses a broad range of models, including linear regression, path analysis, and latent growth curve analysis, as special cases. The fundamental principle behind SEM estimation involves minimizing the discrepancy between the sample correlation matrix and the model-implied correlation matrix. However, many estimation techniques rely heavily on the assumption of multivariate normality, which may not always hold in practical applications [4].

An alternative approach is the empirical likelihood (EL) method, a nonparametric statistical inference technique that avoids strict distributional assumptions [3]. Integrating EL into SEM offers a promising direction for research [5; 6; 7], yet existing studies on this integration remain limited in scope.

This study systematically formulates the theoretical foundation of SEM and illustrates its application with a variety of examples [1]. Furthermore, it examines the EL framework within the SEM context [2], surveying key results from the existing literature. The study concludes by identifying potential research directions and open problems that warrant further investigation.

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ACTA SOCIETATIS MATHEMATICAE LATVIENSIS
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EXISTENCE OF A POSITIVE SOLUTION FOR A SYSTEM OF BOUNDARY VALUE PROBLEMS WITH NONLINEAR BOUNDARY CONDITIONS INVOLVING MULTIPLICATION

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We present a result on the existence of at least one positive solution for a system of two nth-order differential equations

$$x^{(n)}(t) + f_1(t, x(t), y(t)) = 0, \quad t \in (0,1),$$
$$y^{(n)}(t) + f_2(t, x(t), y(t)) = 0, \quad t \in (0,1),$$

endowed with nonlocal nonlinear boundary conditions

$$x'(0) = ... = x^{(n-1)}(0) = 0, \quad x(1) + x'(1) = k_1\, x^{(n-1)}(\alpha)\, x^{(n-1)}(\beta),$$
$$y'(0) = ... = y^{(n-1)}(0) = 0, \quad y(1) + y'(1) = k_2\, y^{(n-1)}(\gamma)\, y^{(n-1)}(\delta).$$

The nonlinearity of the boundary conditions is caused by multiplication of the $(n-1)$th derivatives at interior points. Standard method of obtaining solutions is to rewrite problem as an equivalent system of integral equations and seek solutions as fixed points of the corresponding integral operator, see [1]. The proof of the existence of fixed points relies on the vector version of Krasnosel'skiĭ's fixed point theorem [2].

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ON THE PROJECT DEVOTED TO THE DEVELOPMENT OF A FUZZY LOGIC BASED APPROACH TO THE VALUE OF INFORMATION IN OPTIMAL CONTROL UNDER UNCERTAINTY

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This report will introduce the conference participants to the purpose and objectives of the project *"A fuzzy logic based approach to the value of information estimation in optimal control problems under uncertainty with applications to ecological management"*, which was recently approved by the Latvian Council of Science and will be conducted from January 2025 to December 2027 in the Department of Mathematics at the University of Latvia.

Information has an economic value because it enables decision-makers to make better decisions than they could make without it. The concept of Value of Information (VoI) is based on the information theory and the decision theory and was initially introduced in the late 60s [1; 2] (see also [3]) and later developed in a series of works (e.g., [4; 5]). It constitutes a quantitative instrument to assess decisions under uncertainty related to the procurement of information. Specifically, VoI makes it possible to evaluate the profit from obtaining new information against the background of its current level.

The aim of the project is to develop a rigorous and systematic framework for the fuzzy modeling of uncertainties for decision-making with applications in ecological management. This includes further development of the theory of VoI, analysis of VoI for related optimal control problems, as well as development of decision-making procedures for assessing the need of acquiring new information.

Acknowledgement. This research is funded by the Latvian Council of Science, project No. lzp-2024/1-0188.

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Acta Societatis Mathematicae Latviensis
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ON OSCILLATION OF SOLUTIONS OF THE SECOND ORDER DIFFERENTIAL EQUATIONS

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The theory of oscillators is widely used in engineering, physics and other fields. The simplest oscillator $x'' + k^2 x = 0$ has solutions among the trigonometric functions. More advanced equations of the form $x'' + 2\delta x' + k^2 x = 0$ have solutions of a more complicated behavior, involving also an exponential function. The next level includes the external force, and the equation looks like $x'' + 2\delta x' + k^2 x = h \cos \omega t$. The solutions of this equation are combinations of constants, trigonometric and exponential functions. For some values of parameters the resonance phenomenon can be observed. The situation becomes more complicated after introducing the nonlinear term on the left side. The equation may be of the form

$$x'' + 2\delta x' + k^2 x \pm x^3 = h \cos \omega t. \tag{1}$$

The behavior of solutions can become irregular and even chaotic. The search for the parameters δ, k, h, ω, ensuring the desired behavior, is a challenging problem. Equation (1) can be written as the system of three equations

$$\begin{cases} x' = y, \\ y' = -2\delta x' - k^2 x \mp x^3 + h \cos \omega t, \\ z' = \omega. \end{cases} \tag{2}$$

We are looking for the values of parameters δ, k, h, ω such that the solutions of the equation (1), or, equivalently, the system (2) are sensitive dependent on the initial conditions.

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Acta Societatis Mathematicae Latviensis
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COMPETENCY-BASED EDUCATION IN MATHEMATICS IN LATVIA – CHANGES INTRODUCED BY SCHOOL 2030 AND POSSIBLE IMPROVEMENTS

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In Latvia, as well as around the world, education systems are regularly reviewed and improved, taking into account both current global trends and the development of information technology. The most recent significant changes in Latvia's education system were made within the project School 2030. [1] This was the first time when curricula and teaching approach were revised and changed across all levels of education, from preschool to the grade 12.

Changes were made to subject standards, and new syllabus were created to ensure that schools could provide pupils not only with theoretical knowledge but also with practical skills necessary for today's job market. Special emphasis was placed on critical thinking, problem-solving, collaboration, and digital skills.

New curricula in mathematics in Latvia was introduced in 2018 for elementary school [2] and in 2019 for secondary school [3]. Since the new curricula has been fully implemented for more than four years, and pupils have already taken corresponding exams [4], students who have acquired the new curriculum have started their studies at universities, we can start to analyze how the introduced changes have impacted pupils' and graduates' knowledge and skills. It is essential to understand which aspects of innovations have been successful and which areas still require improvements.

In the presentation we discuss curricula changes introduced by the project School 2030 and possible improvements to mathematics content.

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Acta Societatis Mathematicae Latviensis
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STRUCTURE-PRESERVING DATA-DRIVEN ALGORITHMS

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Structure-preserving data-driven algorithms have recently received great attention, e.g., the development of the symplectic neural networks SympNets [1] and volume-preserving LocSympNets [2] for learning the flow of Hamiltonian and source-free dynamical systems, respectively. It has been demonstrated that structure-preserving neural networks produce qualitatively better long-time predictions. Despite these achievements, learning the flow of a high-dimensional dynamics still poses a great challenge due to the increase in neural network model complexity and, thus, the significant increase in training time [3].

In this work, we propose and study structure-preserving dimensionality reduction techniques in conjunction with structure-preserving data-driven algorithms to enhance computational efficiency, i.e., training and prediction, while at the same time preserving essential properties of the dynamics. To learn the flow of a Hamiltonian dynamics, which can be well modeled in a lower-dimensional subspace, e.g., such as nonlinear localized crystal lattice excitations, also known as discrete breathers, we perform dimensionality reduction by learning the so-called *symplectic lift* and *symplectic projection* matrices of the *proper symplectic decomposition* (PSD) [4]. We find that learning the SPD-reduced Hamiltonian dynamics is not only more computationally efficient than learning the high-dimensional model, but we can also obtain qualitatively good long-time predictions.

Acknowledgement. This research is funded by the Latvian Council of Science, project No. lzp-2024/1-0207.

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Acta Societatis Mathematicae Latviensis
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TWO DIMENSION GENE REGULATORY SYSTEM WITH SEASONALITY

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The authors in [1] summarize the progress on the molecular and genes mechanisms of seasonal regulation and seasonal changes. Authors describe how important are rhythmic cycles and that small changes in seasonality cycle can involve changes in genes.

In the articles [2], [3], [4] the authors study system which is part of the gene regulatory system.

The authors in [5] observed the seasonality function

$$r_0 = r(1 + \epsilon \sin \theta t). \tag{1}$$

In the current research we extend the two dimensional gene regulatory system

$$\begin{cases} \dfrac{dx_1}{dt} = \dfrac{1}{1 + e^{-\mu_1(w_{11}x_1 + w_{12}x_2 - \theta_1)}} - v_1 x_1, \\ \dfrac{dx_2}{dt} = \dfrac{1}{1 + e^{-\mu_2(w_{21}x_1 + w_{22}x_2 - \theta_2)}} - v_2 x_2 \end{cases} \tag{2}$$

by change the parameters w_{12} and w_{21} with the simplified seasonality function

$$S(w_{ij}, t) = F_S = w_{ij} + \sin t. \tag{3}$$

With the seasonality functions is possible to drive the system solutions from one attractor to another one. The examples and reasons of driving solutions from one attractor to another one was provided in the papers [6], [7]. The seasonality functions in some fixed moment t can change types of solutions attractions for the gene regulatory system (2).

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ON TWO SYSTEMS OF DIFFERENCE EQUATIONS WITH MANY PERIODIC SOLUTIONS

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We consider the global behavior of the systems of first order piecewise linear difference equations:

$$\begin{cases} x_{n+1} = |x_n| - y_n - b, \\ y_{n+1} = x_n - |y_n| - d, \end{cases} \quad (1) \qquad \begin{cases} x_{n+1} = |x_n| - y_n - b, \\ y_{n+1} = x_n - |y_n| + d, \end{cases} \quad (2)$$

$n = 0, 1, 2, ..., (x_0, y_0) \in \mathbf{R}^2$, where parameters b and d are any positive real numbers. The special case of (1) with $b = d = 1$ is considered in dissertation of W. Tikjha [2]. The special case of (2) with $b = 3$ and $d = 1$ is considered in [3].

In general case of system (1) there exist an unstable equilibrium $(d; -b)$. It has been shown that there are no solutions with period 2, 3 and 4, but depending on the values of parameters b and d there are solutions with periods 5, 6, 7, 11, 12, 13, 16, 17, 18, 19, 20, 24, 27, 30, 36. The obtained results are published in the article [1].

In general case of system (2), if $2d - b \leq 0$, then there exist an unstable equilibrium $(\frac{-2b-d}{5}; \frac{2d-b}{5})$. It has been shown that there are no solutions with period 2, but depending on the values of parameters b and d there are solutions with periods 3, 4, 7, 10, 11, 14. The results obtained here were created in collaboration with student J.L. Kristone.

Other types of periodic points of (1) and (2) have not been proven, they have not been seen in numerical experiments, but they probably exist. In some cases, several cycles exist simultaneously. We have a hypothesis that all solutions of systems (1) and (2) are periodic or eventually periodic.

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Acta Societatis Mathematicae Latviensis
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MAXIMAL POLYGONS ON A TRIANGLE LATTICE

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The shapes on a triangle grid are known as polyiamonds. A polyiamond is a plane figure consisting of regular unit triangles attached to each other along the sides of a whole length. If the area of a polyiamond with a fixed perimeter is the largest (smallest), it is called a maximal (minimal) polyiamond. Maximal and minimal polygons on a triangular lattice are studied in [1] and [2], where the formula to calculate the area of some lattice polygons is also given.

The maximal polyiamonds considered here are those for which not only the perimeter but also the number of edges m is fixed. For such a figure, we denote the area (number of triangles forming the figure) by $T(p, m)$, where p is the perimeter and m is the number of edges. While in the case $m = 3$ finding the formula for the area of maximal triangles is a trivial task, starting with the quadrilateral formula is not so easy.

In the case $m = 4$, the area of maximal polyiamond can be calculated with formula

$$T(p;4) = \begin{cases} 2 \cdot \text{round}\left(\frac{p}{4}\right)\left(\frac{p}{2} - \text{round}\left(\frac{p}{4}\right)\right) \text{ if } p \text{ is even and } p > 3, \\ \left(\text{round}\left(\frac{3p}{8}\right)\right)^2 - \left(3 \cdot \text{round}\left(\frac{3p}{8}\right) - p\right)^2 \text{ if } p \text{ is odd and } p > 4, p \neq 9, \\ 7 \text{ if } p = 9. \end{cases}$$

Since all maximal polygons on a triangle lattice for a fixed perimeter p, $p \geq 6$, $p \neq 7$, are hexagons, the formula given in [1] is also the formula to find the area of maximal hexagons on a triangle lattice. This maximal polygon area formula can be written as following

$$T(p;6) = \begin{cases} 6k^2 \text{ if } p = 6k, \\ 6k^2 + 2k - 1 \text{ if } p = 6k+1, \\ 6k^2 + 4k \text{ if } p = 6k+2, \\ 6k^2 + 6k + 1 \text{ if } p = 6k+3, \\ 6k^2 + 8k + 2 \text{ if } p = 6k+4, \\ 6k^2 + 10k + 3 \text{ if } p = 6k+5. \end{cases}$$

where p is perimeter and $k \in \mathbb{N}$.

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MEALY MACHINES FROM POLYNOMIALS

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We will consider mapping

$$\mu[f] : g(X) \mapsto f(X)g(X),$$

where $f(X)$ and $g(X)$ are elements of ring $R[[X]]$.

Example 1. What kind of group is determined by polynomial $f(X) = 1 + X + X^2$?

Let $f(X) = 1 + X + X^2$ and $g(X) = s_0 + s_1X + s_2X^2 + \cdots = \sum_{k=0}^{\infty} s_k X^k$, then

$$\begin{aligned} g\mu[f] &= s_0 + (s_0 + s_1)X + (s_0 + s_1 + s_2)X^2 + (s_1 + s_2 + s_3)X^3 + \cdots \\ &= s_0 + (s_0 + s_1)X + \sum_{k=0}^{\infty}(s_k + s_{k+1} + s_{k+2})X^{k+2} \end{aligned}$$

and

$$\begin{aligned} g\mu_r[f] &= r + s_0 + (r + s_0 + s_1)X + (s_0 + s_1 + s_2)X^2 + (s_1 + s_2 + s_3)X^3 + \cdots \\ &= r + s_0 + (r + s_0 + s_1)X + \sum_{k=0}^{\infty}(s_k + s_{k+1} + s_{k+2})X^{k+2}. \end{aligned}$$

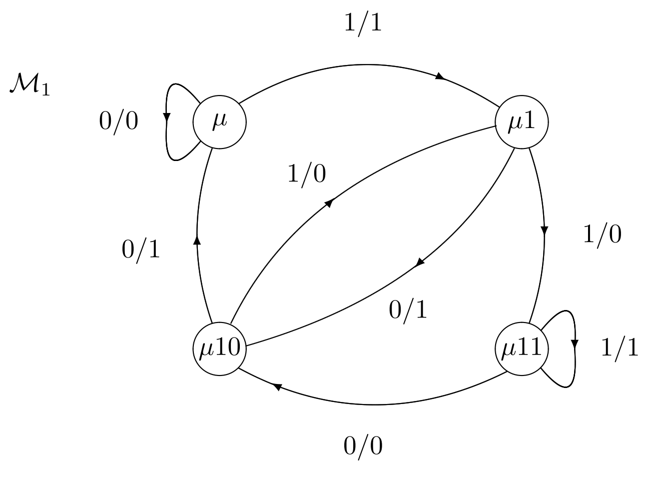

Figure 1. Machine defined by $1 + X + X^2$ in the field $GF(2)$.

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HYERS-ULAM STABILITY OF IMPLICIT VOLTERRA INTEGRAL EQUATIONS ON TIME SCALES

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Consider nonlinear k-th order Volterra integrodifferential equation on an arbitrary time scale $\mathbb{T}$

$$x^{\Delta^k}(t) = f\left(t, x(t), x^{\Delta}(t), \dots, x^{\Delta^k}(t), \int_{t_0}^{t} k(t, s, x(s), x^{\Delta}(s), \dots, x^{\Delta^k}(s))\Delta s\right) \tag{1}$$

with initial conditions

$$x^{\Delta^i}(t_0) = x_i, \quad i = 0, 1, 2, \dots, k-1, \quad t_0, t \in I_{\mathbb{T}} = [t_0, +\infty) \cap \mathbb{T}.$$

We reduce equation (1) to implicit Volterra integral equation

$$z(t) = F\left(t, z(t), \int_{t_0}^{t} K(t, s, z(s))\,\Delta s\right), \quad t_0, t \in I_{\mathbb{T}} = [a, +\infty) \cap \mathbb{T}, \tag{2}$$

where $z\colon I_{\mathbb{T}} \to \mathbb{R}^{n(k+1)}$ is the unknown function, $K\colon I_{\mathbb{T}} \times I_{\mathbb{T}} \times \mathbb{R}^{n(k+1)} \to \mathbb{R}^{n(k+1)}$ be rd-continuous in its first and second variable, $L\colon I_{\mathbb{T}} \to \mathbb{R}$ be rd-continuous, $\gamma > 1$, $\beta = L(s)\gamma$,

$$|K(t, s, z) - K(t, s, z')| \le L(s)|z - z'|, \quad z, z' \in \mathbb{R}^{n(k+1)}, \quad s < t,$$

$$|F(t, z, w) - F(t, z', w')| \le M(|z - z'| + |w - w'|),\; m = \sup_{t \in I_{\mathbb{T}}} \frac{1}{e_\beta(t, t_0)}\left|F(t, 0, \int_{t_0}^{t} K(t, s, 0)\Delta s)\right| < \infty.$$

If $M(1 + 1/\gamma) < 1$, then the integral equation (2) has a unique solution $z \in C^k_\beta(I_{\mathbb{T}}; \mathbb{R}^{n(k+1)})$, where $C^k_\beta(I_{\mathbb{T}}; \mathbb{R}^{n(k+1)})$ be the Banach space of rd-continuous functions such that

$$\sup_{t \in I_{\mathbb{T}}} \frac{\max_{0 \le i \le k} |x^{\Delta^i}(t)|}{e_\beta(t, t_0)} < \infty.$$

We also prove that equation (1) is Hyers-Ulam stable.

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ON SOME EXTREMUM PROBLEMS IN COMBINATORIAL GEOMETRY

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Combinatorial geometry has many new non-trivial problems related to the determination of extreme values. Some of them will be formulated and ideas for solving them will be discussed. One example is a modification of the classical isoperimetric problem, where instead of polygons, one considers polyominoes or polyiamonds with both a fixed perimeter and a fixed number of edges. Let $T(p, m)$ denote the maximum number of triangles, that can be contained by a polyiamond whose perimeter and number of sides are p and m, respectively. We note that starting from $m = 7$, the maximal polygons are not convex, and in the general case the determination of the numbers $T(p, m)$ has not been studied to our knowledge.

The next example: how to find a shape that can be assembled in as many ways as possible from the shapes of a given set?

The topic of shape exclusion (blocking) is one of the richest sources of extreme problems, ranging from various mathematical competitions and research to unsolved problems. Typical problems in this field are: what is the minimum number of shapes F that must be placed in square $n \times n$ to no more be able to fit a shape G. Clearly, more research has been done on problems where the simplest shapes, the unit square and the domino, play the role of F. Non-trivial results and surprises are already contained in the problem of using the simplest shapes, namely $F = G = 1 \times 2$ (domino), see [1]. Interestingly, the analysis of the case $F = 1 \times 2$, $G = 2 \times 2$, is considerably simpler than when $G = 1 \times 2$. As far as we know, the first book in which one can find the tasks of excluding shapes (namely, pentominoes by monominoes (or unit squares) on a chessboard 8×8) is the Golomb's classic book [2]. Exclusion problems in other areas (graph theory, statistical physics, percolation theory) may have to do with the following concepts: matching, minimum dominating sets, domination number, square grid graphs, dimmers, and others [3].

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CERTAIN PARTIAL ORDERS ON STRONG RICKART RINGS

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A ring R is said to be a *Rickart ring* if, for every $a \in R$, there exist idempotents $e, f \in R$ such that, for all $x \in R$,

$$ax = 0 \text{ iff } ex = x \tag{1}$$

and

$$xa = 0 \text{ iff } xf = x. \tag{2}$$

If the ring R is involutive and, for every $a \in R$, the idempotents e and f from conditions (1) and (2) are furthermore self-adjoined, then R is said to be a *Rickart *-ring.* A typical example of a Rickart *-ring is the ring of bounded operators on a Hilbert space.

This talk is concerned with non-involutive generalizations of Rickart *-rings called *strong Rickart rings.* This approach allows for the transfer of knowlegde from the field of Rickart *-rings to a wider class of rings without involution, showing that many notions can be defined and described in terms that are independent of an involution and that the results obtained for Rickart *-rings can often be at least partly preserved.

In this talk, we focus on conditions for the existence of meets and joins under several partial orders on strong Rickart rings. We deal with the diamond order, the strong right star order, the weak right star order and the star order.

For the latter two orders, we find some sufficient conditions for the existence of joins involving certain notions of coherence which were introduced by different authors (see [1] and [3])

We also present a condition for the existence of the join under the diamond order and we show how the existence of the meet and join of two elements of a right-strong Rickart ring under the strong right star order is related to the existence of meet and join under the weak right star order.

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MEASURES ON WEAK BCK-ALGEBRAS

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Our work is within the area of mathematics that uses structures of universal algebra to describe models of logics. The illustrating example is the commonly known correspondence between Boolean algebras and models of propositional logics. The structure of a BCK-algebra was defined by Imai and Iseki [1] in 1966 as an algebraic characterization of logics with a connectivity that behaves like the implication in the propositional calculus. BCK-algebras are a special case (as subreducts) of the broader class of residuated structures. Among residuated structures, so-called relative pseudocomplemented semilattices play a prominent role as they characterize an implication in the intuitionistic logic. To provide a generalization for the case when the intuitionistic logic is non-distributive, Chajda [2] introduced the structure of a sectionally pseudocomplemented lattice. Description of the implication in a sectionally pseudocomplemented lattice led to the definition of weak BCK-algebras by Cīrulis [3]. Concretely, a *weak BCK-algebra* is an algebra $(A, \smallsetminus, 0)$ equipped with a partial order $\leq$ such that 0 is the least element and $\smallsetminus$ is a binary operation on A satisfying $x \leq y$ if and only if $x \smallsetminus y = 0$, and if $x \smallsetminus y \leq z$, then $x \smallsetminus z \leq y$ for all $x, y, z \in A$.

Our work is focused on states and measures on weak BCK-algebras. States can be seen as counterparts of logical evaluation, i.e., on Boolean algebras, they are homomorphisms of a given Boolean algebra to the two-elements Boolean algebra (containing 0 and 1). On more general structures, the range is often taken as real interval $[0, 1]$, as the logical evaluation does not have to be sharp. Measures are generalizations of states that need not preserve the top element 1. In our work, we introduce the notion of states and measures on weak BCK-algebras. We show that algebraic properties which are typical for measures and states on other structures are satisfied also in the case of weak BCK-algebras. We investigate the connection of measures with congruences, with the stronger notion of measure morphisms and we relate these notions to basic algebras and Rickart rings.

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OPTIMIZATION OF ADMINISTRATIVE DIVISION AS A GRAPH THEORY PROBLEM

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Efficient and equitable access to municipal services hinges on well-designed administrative divisions. It requires ongoing adaptation to changing demographics, infrastructure, and economic factors. We propose a novel data-driven method for territorial division based on the Voronoi partition of edge-weighted road graphs and the vertex k-center problem, a special case of the minimax facility location problem. By considering road network structure and strategic placement of administrative centers, this method seeks to minimize travel time disparities and ensure a more balanced distribution of administrative time burden for the population. We show implementations of this approach in the context of Latvia.

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ON DUFFING EQUATION

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Properties of solutions to the Duffing equation are considered. These properties relate to the notion of the index of a solution. We show that indexes of solutions change in a non-monotone way.

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MATHEMATICAL MODEL OF MICROWAVE PRE-TREATED BIOMASS PYROLYSIS DEPENDING ON BIOMASS POROSITY

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To model microwave pre-treatment of biomass, we define reactions for biomass organic compounds that in the thermic process decompose into volatiles and carbon [2]. Reactivity changes of microwave pre-treatment of biomass we describe using porosity. Coal reaction with oxygen described with equation

$$C + \theta O_2 \to 2(1-\theta)CO + (2\theta - 1)CO_2, \tag{1}$$

where $\theta \in [0.5; 1]$. Chemical reactions are modeled using Arrhenius equation

$$X' = A \cdot X \cdot e^{-\frac{E_X}{RT}}. \tag{2}$$

Gases in model are described using the Darcy law, mass balance equation and ideal gas law in this system [1]:

$$\begin{cases} \overrightarrow{v} = \frac{\alpha}{\mu}(\overrightarrow{g}\rho - \nabla p), \\ (\gamma\rho)' + div(\rho\overrightarrow{v}) = q, \\ pM = \rho RT. \end{cases} \tag{3}$$

The numerical solution was found in program MatLab using finite volume method and finite deference scheme.

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60 YEARS OF FUZZY SETS

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Since L.A. Zadeh introduced the concept of fuzzy sets in 1965 [1], the field has evolved into a vibrant area of research, bridging traditional mathematical theories with the study of uncertainty, vagueness, and imprecision. Early milestones in this integration included the development of fuzzy topologies by C.L. Chang [2], fuzzy algebraic structures by A. Rosenfeld [3], fuzzy category theory by A. Šostak [4], and fuzzy metric spaces by I. Kramosil and J. Michalek [5]. These foundational contributions have established a solid theoretical basis for fuzzy mathematics, inspiring researchers to explore new paradigms and extend classical constructs into the fuzzy context.

Beyond its theoretical advancements, the practical utility of fuzzy sets has been demonstrated across a wide range of applications, including natural language processing [6], pattern recognition [7], decision-making, and optimization problems [8]. Fuzzy logic and systems have become indispensable tools in addressing real-world problems where classical methods fall short due to inherent uncertainty or imprecision.

Here we would like to reflect on 60 years of progress in fuzzy sets, celebrating their profound impact on both mathematics and applications, and using an opportunity to discuss current trends and envision future directions in this ever-evolving field, emphasizing its relevance in tackling the challenges of a complex and uncertain world.

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DEVELOPMENT OF STRUCTURE-DRIVEN AND DATA-DRIVEN METHODS FOR ANALYSIS AND CONTROL OF COMPLEX DYNAMICAL SYSTEMS

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This presentation aims to give an overview of the planned research within the framework of the project *"Development of structure- and data-driven methods for analysis and control of complex dynamical systems"*, which was recently approved by the Latvian Council of Science and will be conducted from January 2025 to December 2027 in the Department of Mathematics at the University of Latvia.

Recent decades have been marked by fundamental transformations that created enormous opportunities in different fields of technology and science. One of the signs of these transformations is the overwhelming volume of data produced. Leveraging this data is the key to the next-generation technologies. While there has been substantial progress in developing advanced machine learning algorithms, the use of mathematical structures, inherent to many technical and technological processes can not only substantially increase the quality of the used algorithms, but also provide additional insight and give an impetus to the further development of data-driven methods. This project aims to contribute by developing novel, data- and structure-driven approaches for the modeling of complex technical systems and using the obtained models for designing the control laws. The novelty of the project is twofold:

1. a unique interplay between the analytical, differential-geometric methods and elaborated numerical schemes, which are consistent with the underlying geometrical structure of the systems under study;
2. a systematic development and joint application of theoretical, Koopman operator-based, and numerical, machine learning-based methods within the control-theoretic framework.

The presentation will be centered on three key aspects of the project: data-driven methods, structured dynamical systems, and control applications [1; 2; 3]. We will outline the strengths and limitations of current approaches and highlight potential opportunities for further developing the research field.

Acknowledgement. This research is funded by the Latvian Council of Science, project No. lzp-2024/1-0207.

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ON THE CHOICE BETWEEN COOPERATION AND COMPETITION IN DIFFERENTIAL GAMES

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This study examines the dynamics of cooperation in a differential game, focusing on the profitability of forming coalitions compared to competing individually against other players. The benefits of coalition formation are assessed using the Value of Cooperation (VC) and the Normalized Value of Cooperation (NVC) [1; 2]. Coalition strength is evaluated through the characteristic function, developed using four distinct methods [3; 4]. The research also introduces the Value of Confrontation (VCF) and the Normalized Value of Confrontation, applicable to both individual players and coalitions.

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RUNGE-KUTTA METHOD AND SINGULAR SPECTRAL ANALYSIS

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Runge-Kutta methods were developed at the beginning of 1900-ies by German mathematicians Carl Runge and Wilhelm Kutta as a family of iterative methods for differential equations solutions. In the dynamic systems, modelling Runge-Kutta method is the one of most popular methods for numeric analysis of differential equations.

The main idea of the SSA was formulated at the end of the 1940-ies by Kari Karhunen and Michel Loève. Karhunen-Loève theorem describes how to extract signal from the mixture "signal plus noise" and first 50 years of its existence SSA was used as instrument for signal cleaning from the noise. In 2001 N. Golyandina, V. Nekrutkin, and A. Zhigljavsky find that it is possible to write linear recurrent form LRF for signal restored by SSA

$$f_n = a_1 f_{n-1} + \cdots + a_d f_{n-d},$$

which gives the possibility to calculate any time series value using previous d values. Using this formula for last d time series values, it gives the possibility to calculate value out of initial definition area of time series, providing the possibility to forecast time series values.

For time series generated by solutions of differential equations, it is possible to use SSA to extrapolate time series and compare extrapolation with numeric solution of differential equation. We investigate relations between LRF, received using SSA, and Runge-Kutta method. Results are discussed.

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ON THE YONEDA LEMMA IN FUZZY CATEGORIES

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The Yoneda lemma is a cornerstone of category theory, providing a universal insight into the relationship between objects and morphisms in a category. In enriched category theory, where hom-sets are replaced by structures such as monoids, topological spaces, or lattices, the Yoneda lemma admits generalizations that preserve its essential spirit while adapting to enriched contexts. The enriched Yoneda lemma can be stated for closed symmetrical monoidal categories [2] or even generalized further [1].

Enriching categories over the closed symmetrical monoidal category of L-sets [3], we effectively assign each morphism in a category a membership degree. Since L-set enriched categories naturally embed into fuzzy categories – where we additionally can assign a membership degree to objects – there arises a compelling motivation to investigate a formulation of the Yoneda lemma in fuzzy categories. An analogue of the Yoneda lemma for fuzzy categories would establish representability in settings where classical or crisp structures are insufficient.

In this talk, we will explore how both the weak and strong forms of the enriched Yoneda lemma could be generalized to fuzzy categories. Furthermore, we will examine functor representations into the crisp category $\boldsymbol{L}$-**Set** and the fuzzy category $\boldsymbol{FL}$-**Set**.

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Acta Societatis Mathematicae Latviensis
Abstracts of the 13th Latvian Mathematical Conference, January 24-25, 2025, Riga, Latvia

DATA DRIVEN LEARNING OF PROCESSING METHODS FOR HAMILTONIAN DYNAMICS

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Classically, symplectic numerical methods are used when predicting Hamiltonian dynamics. More recently, symplectic neural networks, such as SympNets [1], have been proven as viable alternatives by learning Hamiltonian dynamics directly from data. Each approach has its advantages – the error and stability of numerical methods can be controlled through factors such as the time-step h, whereas neural networks eliminate the need for explicit knowledge of the Hamiltonian equations. Unlike classical numerical methods, SympNets lack guarantees regarding prediction precision, as error analysis cannot yet be rigorously performed.

In this work, we propose a novel approach using a processing method. Unlike the standard approach, where pre-processing and post-processing mappings are predefined to either increase the order of the kernel method or reduce its error [2], we replace these mappings with ones learned by SympNets. A key contribution of this work is Theorem 1, which establishes the theoretical foundation for our method [3].

Theorem 1. *Let K_h be a numerical method of order p, and let χ_h be a SympNets neural network of order k, where h is the time-step and N is the number of steps. Then, the resulting processing method*

$$\hat{K}_h^N = \chi_h \circ K_h^N \circ \chi_h^{-1}$$

will be of order of at least $\min(k, p)$.

The processing method incorporates neural networks while retaining the ability to control errors through the choice of time-step h. Moreover, thanks to the construction and Theorem 1, familiar error analysis techniques from classical numerical analysis remain applicable.

Acknowledgement. This research is funded by the Latvian Council of Science, project No. lzp-2024/1-0207.

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Abstracts of the 13^{th} Latvian Mathematical Conference, January 24-25, 2025, Riga, Latvia

WEAKLY NONLINEAR INSTABILITY: NUMERICAL ASPECTS

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Weakly nonlinear theory is often used to describe the development of instability beyond the threshold where the base flow becomes linearly unstable [1]. The main steps of the procedure are as follows. First, linear stability problem is solved and critical values of the parameters of the problem are evaluated numerically. Second, method of multiple scales is applied in the neighbourhood of the critical point where the value of the Reynolds number (or other parameter characterizing the flow) is slightly above the critical value. Multiple scale expansion up to order three is then considered where the small parameter ε represents the relative increase of the Reynolds number above the critical value. At order one in ε linear stability problem is recovered. Three boundary value problems for ordinary differential equations (one of which is resonantly forced) have to be solved at order two in ε. Solvability condition has to be used at each order in ε. In particular, solvability condition at order two allows to determine the group velocity. Finally, using solvability condition at order three we obtain the amplitude evolution equation for the most unstable mode. In this case it is the complex Ginzburg-Landau equation [2] of the form:

$$\frac{\partial A}{\partial \tau} = \sigma A + \delta \frac{\partial^2 A}{\partial \xi^2} - \mu A|A|^2, \tag{1}$$

where σ, δ and μ are complex coefficients.

In the present paper we describe numerical aspects of this procedure. Linear stability problem is solved using collocation method based on Chebyshev polynomials. Similar approach is used to solve two out of three boundary value problems derived at order two in ε. One of the three boundary value problems is resonantly forced so that the method of singular value decomposition is used to solve it. The coefficients σ, δ and μ in (1) are expressed in terms of definite integrals containing the solutions at orders one and two in ε. In addition, some of the terms contain derivatives of order three so that accurate numerical method is needed to evaluate these derivatives. Chebyshev collocation method is used in the paper for this purpose.

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Abstracts of the 13th Latvian Mathematical Conference, January 24-25, 2025, Riga, Latvia

MATHEMATICAL THINKING AND SKILLS - A KEY IN THE STUDIES OF FUTURE SMART ELECTRONICS AND DATA TRANSMISSION ENGINEERS

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High-level mathematical skills are indispensable in engineering education in order to understand various simple and also complex engineering problems and find solutions to them, as well as to promote innovation in the engineering science. Mathematics can be safely called the alphabet of the engineering language.

In order for future engineers to have such skills, it is necessary to have stable foundation in this field. The lack of strong knowledge of the most important topics will not allow to fully master the topics of higher mathematics. More than 50% of students do not have this knowledge or are at a very low level when starting their studies.

In order not to encounter problems of this nature, there are continuous reforms and changes both in the education system of Latvia and elsewhere in the world. These processes and their impact on growth of youth are the subject of discussions, even to protests. One of the most widely discussed blocks of learning subjects is exact sciences, their necessity, quality and intensity of teaching (from various aspects) [1].

Since it is currently possible to study mathematics at three different levels in secondary schools, it could be assumed that only young people who have mastered Mathematics II (the highest level) should study engineering. But this situation is not so unambiguous. A sufficiently large number of schools do not offer to study the subject Mathematics II at all. Students have to make choice to study only Math I or also Math II already at the beginning of the 10th grade, not to mention the situation in vocational schools. So, the situation when a future engineering student is a person who has graduated from any of the 3 models is unequivocally correct.

In order to more thoroughly study the current situation, in January 2024, the survey of first to fourth year students of RTU Smart Electronics, Telecommunications and Telematics was organized. In the survey the author of the work tried to find out the assessment of the learning process and content in mathematics from the point of view of students, compare it with the results of student work during studies and growth.

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MODELING OF DATA COMPARISON BY MEANS OF OBJECT ORIENTED AND PROPERTY ORIENTED CONCEPT LATTICES

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The formal concept analysis introduced by B. Ganter and R. Wille [1] has been originally used for development of theory for analysis of objects and their properties. As part of the formal concept analysis the formal context is defined as a triple (X, Y, R) where X is a set, whose elements are interpreted as objects, Y is a set, whose elements are interpreted as properties and $R \subseteq X \times Y$ is a relation where the entry xRy is interpreted as "an element x has property y". Traditional theory of formal concept analysis deals with crisp sets and relations between object and properties which are interpreted in such way that either the object has or does not have respective property.

As part of our research (A. Šostak et al. [2]) we introduced fuzzy preconcept lattices to investigate the fuzzy case for relations between objects and their properties. We define a pair of sets X and Y and a fuzzy relation $R : X \times Y \to L$. Now the set X is interpreted as the set of some objects and the set Y as the set of some properties, the value $R(x, y) = \alpha \in L$ means that the object x is related to the property y to the degree α. The quadruple (X, Y, R, L) is called a formal context. According to definition given a formal fuzzy context (X, Y, R, L), a pair $P = (A, B) \in L^X \times L^Y$ is called a fuzzy preconcept. Furthermore we analyzed the object oriented and property oriented concept lattices in [3]. In our research $L = (L, \leq, \wedge, \vee)$ denotes a complete lattice, that is a lattice in which joins $\bigvee M$ and meets $\bigwedge M$ of all subsets $M \subseteq L$ exist. An object oriented fuzzy concept is defined as the pair (A, B) such that $A = B^{\blacklozenge}$ and $A^{\square} = B$. Similarly, a property oriented fuzzy concept is defined as a pair $(A, B) \in L^X \times L^Y$ such that $A = B^{\blacksquare}$ and $A^{\lozenge} = B$.

We extend our current research to the modeling of data comparison using set X containing certain data (e.g., names together with several attributes characterizing these names) and set Y containing similar data (e.g., similar names and attributes). Such modeling is practicable at least for two purposes – matching the given data with certain external data or comparing the data inside a given data set to search for similar or duplicate entries.

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Acta Societatis Mathematicae Latviensis
Abstracts of the 13$^{\text{th}}$ Latvian Mathematical Conference, January 24-25, 2025, Riga, Latvia

FLAME SHAPING BASED ON DIFFERENTIAL PHYSICS SIMULATIONS WITH NEURAL NETWORK TRAINING

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This work presents the method for shaping flames using differentiable physics-based simulations, advancing traditional fire control techniques by enabling customizable flame shapes in real-time. This work builds on previous work [1] and further extends and develops the research.

This approach integrates combustion dynamics utilizing differentiable physics (DP) to optimize flame projections. The simulation framework relies on the PhiFlow [2] model, which applies the Navier-Stokes equations for fluid dynamics, incorporating velocity and temperature fields controlled by machine learning algorithms. A simplified hot air model is first introduced to understand flow behavior, with controlled nozzles at the bottom emitting hot air jets, which are optimized via gradient-based training to form desired structures. This model was improved to represent real-world fire flames accurately. Including heat release and radiation effects, resulting in more accurate flame simulations that reflect real-world behaviors of stage-flame devices. The governing equation for the combustion is:

$$\rho C_p \left(\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T \right) = \nabla \cdot (\lambda \nabla T) + \dot{\omega}_T + \sigma \epsilon \left(T^4 - T_\infty^4 \right), \tag{1}$$

where ρ is the density of the fluid, C_p is the specific heat capacity, T is the temperature and $\mathbf{u}$ is the velocity field. The term $\nabla \cdot (\lambda \nabla T)$ represents heat conduction, $\dot{\omega}_T$ accounts for the heat released by combustion, and the final term $\sigma\epsilon(T^4 - T\infty^4)$ models radiative heat loss, with σ being the Stefan-Boltzmann constant and T_∞ the ambient temperature. The training used images from a video filmed of flames from a real stage-flames. Loss function is based on the mean squared error between simulated and observed flame shapes. The training is performed using the Adam optimizer, with the loss function designed to minimize the discrepancy between observed and simulated flame behavior.

The work also addresses the limitations of 2D simulations, outlining future steps towards 3D modeling to improve the fidelity of flame behavior. The integration of multiple flame sources and further exploration of alternative fuel types holds the potential to expand the creative possibilities for fire-based visual effects. This research is based on the developmentof a basic system of heat conduction equations, however, the work is worthy of application, for which mathematical analysis is also planned in the future, which could validate the trained neural network parameters.

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Acta Societatis Mathematicae Latviensis
Abstracts of the 13th Latvian Mathematical Conference, January 24-25, 2025, Riga, Latvia

FLOW-DRIVEN PARTICLE DEPOSITION FOR SIMULATION OF BIOMASS EXTRACTION

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This study extends previous research [1] on biomass extraction from diverse plant materials by introducing dynamic effects due to particle deposition. The framework moves beyond assumptions of static particles, incorporating particle motion via modified equations in Lagrangian coordinates [2] and potentially time dependent extractor geometries. These enhancements can allow to capture the intricate dynamics of solvent flow and its interactions with plant material. While prior work focused on supercritical carbon dioxide (scCO2) based extraction, this study emphasizes the possibility to broaden the scope to include other solvent-based extractions and potential applications in filtration and wood processing, among others.

The coupled system of equations integrates fluid dynamics for solvent flow, reaction-advection-diffusion model for solute transport, and equations for remaining solute concentration in the biomass. The transfer of active material between the solid and fluid phases is governed by the Langmuir isotherm. Using finite volume techniques implemented in the Octave/Matlab environment, the model simulates the temporal evolution of solute distribution and solvent velocity in two and three dimensions. This modular framework can accommodate custom laws to represent diverse plant materials, offering versatility across industrial contexts with varied extraction mediums.

Simulation results analyze the performance of the enhanced model, highlighting its strengths and limitations. This research contributes to optimizing extraction processes, providing valuable insights for industries such as functional foods, nutraceuticals, pharmaceuticals, cosmetics, filtration, and wood processing.

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Abstracts of the 13th Latvian Mathematical Conference, January 24-25, 2025, Riga, Latvia

ADVANCED DATA SCIENCE METHODS IN MEDICAL RESEARCH

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In this talk, solutions to common statistical challenges in medical and biostatistical studies are explored. It will be highlighted how advanced methods can improve analyses in both standard and non-standard study designs, moving beyond traditional approaches.

One example focuses on using empirical likelihood, a nonparametric method introduced in 1988, for two-sample comparisons. We discuss comparing two samples graphically by adding confidence intervals or simultaneous confidence bounds, which is done using two-sample location-scale models. Such an approach provides a more detailed analysis of discrepancies between two samples [1]. A method with a similar theoretical background was developed and applied to normalize laboratory data measured on separate plates [2].

Another example is developed in the context of survival analysis. We demonstrate an approach for various two-sample problems using the empirical likelihood method. This method performs slightly better for small samples in comparison to the existing ones; however, it demonstrates substantially better computational efficiency compared to other empirical likelihood-based alternatives.

We also provide a brief overview of data problems that were solved using data-driven machine learning techniques (such as clustering) regarding the Type 1 diabetes patients data.

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Abstracts of the 13th Latvian Mathematical Conference, January 24-25, 2025, Riga, Latvia

A FRESH VIEW ON STAR TIME SERIES MODELS WITH DIFFERENT TRANSITION FUNCTIONS

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Smooth transition autoregressive (STAR) time series models are particularly useful when dealing with data that contain structural breaks. These models, with varying transition functions and threshold variables, are tailored to time series with different patterns of structural changes.

The novelty of this work includes proposing a new asymmetric smooth transition function, ASTAR, as well as adjusting the estimation procedure of the logistic smooth transition model (LSTAR) in R for the introduced ASTAR and other known transition functions like ESTAR.

The structure of STAR models is given by equation $Y_t = \alpha_0 + \alpha_1 Y_{t-1} + \cdots + \alpha_{p_1} Y_{t-p_1} + \Theta(\gamma, X, c)\left[\beta_0 + \beta_1 Y_{t-1} + \cdots + \beta_{p_2} Y_{t-p_2}\right] + \epsilon_t$, where Y_t is a dependent variable (the studied time series), α_i are coefficients in the first regime, $\alpha_i + \beta_i$ are coefficients in the second regime, $\Theta(\gamma, Y, c)$ is a nonlinear transition function having values from [0; 1], γ is a smoothness parameter(s), X is a threshold variable (like Y_{t-1}, ΔY, t, ...), and c is a threshold(s).

Traditional threshold functions are symmetric to the left and to the right from a threshold (ESTAR and LSTAR), or the midpoint between the two thresholds (LSTAR2). However, in practice, transition from one regime to the other sometimes happens faster or slower than returning to the initial regime. Therefore, a transition function with different smoothness parameters may be useful.

For this, I propose the following transition function $\Theta(\gamma_1, \gamma_2, X, c) = 1 - 1.5(1 + 0.5e^{-\gamma_1(X-c)} + 0.5e^{\gamma_2(X-c)})^{-1}$. It has two positive smoothness parameters, γ_1 and γ_2 . If $\gamma_1 > \gamma_2$, the first transition is faster, the second is slower and vice versa if $\gamma_1 < \gamma_2$.

The chosen STAR model is usually estimated with conditional least squares. Conditional upon the parameters of the transition function (γ, c), the estimates of coefficients in the regimes can be estimated by Ordinary Least Squares. The parameters (γ, c) are obtained using grid search by minimizing the residual variance (2-dimensional for LSTAR and ESTAR, 3-dimensional for LSTAR2 and ASTAR). Nonlinear optimization procedure used to minimize the residual variance is the BFGS (Broyden-Fletcher-Goldfarb-Shanno) algorithm. Estimation and its difficulties are described by T. Terasvirta in [1]. Adjustment of estimation procedure of LSTAR model in R for the second order logistic smooth transition model LSTAR2 is presented in [2].

The analysis of suitability of different STAR models for data with different patterns of structural breaks is demonstrated using Latvia's employment data. The results indicate that ASTAR provides a better fit for the employment growth than traditional STAR models.

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NOTES ON SPECTRA OF SOME FUČÍK TYPE PROBLEMS WITH NONLOCAL TWO-POINT BOUNDARY CONDITIONS

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Let us consider the Fučík problem

$$x'' = -\mu x^+ + \lambda x^-, \tag{1}$$

with nonlocal two-point boundary conditions of four types

$$x(0) = 0, \quad x(1) = \gamma x(\xi), \tag{2}$$
$$x'(0) = 0, \quad x(1) = \gamma x(\xi), \tag{3}$$
$$x(0) = 0, \quad x(1) = \gamma x'(\xi), \tag{4}$$
$$x'(0) = 0, \quad x(1) = \gamma x'(\xi), \tag{5}$$

where $\xi = \frac{m}{n} \in (0,1)$ and $\gamma \in \mathbb{R}$, m and n $(0 < m < n)$ are positive coprime integer numbers.

The aim of the study is to identify the main properties of the nonlocal problem spectra and compare them with classical Fučík spectrum. The idea of boundary conditions was taken from the work [1], where the linear Sturm-Liouville equation $x'' = -\lambda x$ was analyzed with boundary conditions (2) - (5).

The obtained results generalize and continue of author's previous established investigations [2], [3], [4].

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Abstracts of the 13$^{\text{th}}$ Latvian Mathematical Conference, January 24-25, 2025, Riga, Latvia

MULTIPLE SAMPLE STATISTICAL INFERENCE USING THE SMOOTHLY TRIMMED MEAN

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The classical trimmed mean is a robust statistic that is primarily used in cases of outliers in data. Let $X_{(1)} \leq ... \leq X_{(n)}$ denote the order statistics of the sample $X_1, ..., X_n$.

Definition 1. [1] The sample α-trimmed mean is defined as

$$\overline{X}_\alpha = \frac{1}{n-r} \sum_{i=r+1}^{n-r} X_{(i)}, \tag{1}$$

where $0 \leq \alpha < 0.5$ is the trimming proportion and $r = \lfloor n\alpha \rfloor$. S.M. Stigler [1] demonstrated that the asymptotic normality of the trimmed mean fails when trimming is done close to break points of the data. As an alternative he proposed that the smoothly trimmed mean should be used instead.

Definition 2. [1] Let $J(u)$ be an appropriate weight function, then the sample smoothly trimmed mean is be defined as

$$\overline{X}_{ST} = \frac{1}{n} \sum_{i=1}^{n} J\Big(\frac{1}{n+1}\Big) X_{(i)}. \tag{2}$$

In the paper [2] the asymptotic properties of this estimator are discussed. Furthermore in [3], a more general version of the weight function $J(u)$ is introduced, the variance of $\overline{X}_{ST}$ expressed, the empirical likelihood method applied to the estimator and a simulation study is conducted showing the advantage of the smoothly trimmed mean.

In this paper we further extend the results from [3] to the two-sample and ANOVA cases by introducing t and F statistics based on the smoothly trimmed mean and by establishing empirical likelihood method for these cases. Finally we compare all the methods introduced through a simulation study using the empirical coverage accuracy and empirical level.

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Abstracts of the 13$^{\text{th}}$ Latvian Mathematical Conference, January 24-25, 2025, Riga, Latvia

A PRIORI ESTIMATE, EXISTENCE, AND UNIQUENESS OF SOLUTIONS FOR A FOURTH-ORDER NONLOCAL BOUNDARY VALUE PROBLEM

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We study a priori estimate, existence, and uniqueness of solutions for a fourth-order boundary value problem with three-point conditions. The main tool in the proof of our existence result is Leray-Schauder continuation principle. Examples are included to illustrate the applicability of the results.

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IN MEMORIAM: ON PROFESSOR ALEKSANDRS ŠOSTAKS CONTRIBUTIONS TO DEVELOPMENT OF FUZZY LOGIC-BASED MATHEMATICAL STRUCTURES

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The talk will be dedicated to academician and professor Aleksandrs Šostaks, honoring his remarkable contributions to mathematics, particularly in the development of mathematics in Latvia. We will briefly recall the foundation of the Latvian Mathematical Society, highlighting professor Šostaks' significant role as one of the founders in 1993 and as its chairman from 1996 to 2001. We will explore Aleksandrs Šostaks' main research areas: fuzzy logic-based topology, bornology, morphology, metrics, concept lattices, fuzzy functions, fuzzy relations and categories, emphasizing the lasting legacy he has left to the mathematical community. In addition to reflecting on the development of the theory of fuzzy logic-based structures through professor Šostaks' work, we will also outline future research directions continued by his former students.

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RECENT RESEARCH AT THE STATISTICAL RESEARCH AND DATA ANALYSIS LABORATORY

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We present the recent research conducted at the Laboratory of Statistical Research and Data Analysis, University of Latvia. The research can be divided into two parts: 1) fundamental research in the area of mathematical statistics; 2) research related to interdisciplinary research projects.

Our main research interest is the development of new nonparametric and robust statistical methods, mainly related to the Empirical Likelihood (EL) method.

The new EL-based methods developed can be categorised into three main groups: 1) robust inference methods; 2) change-point detection methods for time series data; 3) two and more sample inference methods for independent, dependent and censored data. Some more details and future work on these topics will be discussed during the talk.

Although EL-based methods have many advantages over classical procedures such as the classical t-test, the Wilcoxon test, the ANOVA procedure, most of these methods have only been developed in the last few decades and are not implemented in programs such as R, Python or Matlab. However we will discuss some interdisciplinary projects mainly related to medical data analysis where our methods lately are being applied.

It is important to note that our developed methods, procedures and algorithms are implemented in the R package "EL", which deals with comparison of means, medians and other quantiles in the two-sample case; quantile-quantile, probability-probability plots and ROC curves and several other two-sample problems.

To make EL methods more accessible to other scientists, one of our future goals is to introduce two new R packages: EL for robust inference and EL for quantile inference which also will be discussed.

Acta Societatis Mathematicae Latviensis
Abstracts of the 13$^{\text{th}}$ Latvian Mathematical Conference, January 24-25, 2025, Riga, Latvia

ON BEHAVIOUR OF MORPHOLOGICAL OPERATORS IN L-FUZZY GROUPS

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Mathematical morphology originated in the 1970s through the work of G. Matheron and J. Serra, initially motivated by practical geology needs. Over time, it gained significant relevance in other fields, particularly image processing, and is now widely researched and applied. Meanwhile, mathematical morphology theory expanded to the development of a theoretical framework within fuzzy set theory, which was first introduced in the paper by De Baets et al. [1]. There, similar to the "classical" approach, the fuzzy mathematical morphology was based on two operators – dilation $\mathcal{D}$ and erosion $\mathcal{E}$ are induced on the linear space by a fuzzy structuring element S.

Extensive research has been done in the study of fuzzy morphological operators on Euclidean space, leading to discovery of many applications in different practical problems. Taking as a basis a pair of operators that behave like dilation and erosion leads to the development of various abstract, in particular, algebraic frameworks for mathematical morphology. This, in turn, made it possible to develop a categorical view of mathematical morphology, see, e.g., [2], [3], [4] et al. However, a notable limitation of these abstract approaches emanates from the fact that the role of the structural element is practically lost.

As far as we know, there was not much work done to develop "classical"(fuzzy) mathematical morphology theory within the framework of category theory. This in turn essentially restricts the possibility of considering operations between (fuzzy) morphological spaces, their transformations, products, and direct sums of (fuzzy) morphological spaces, etc.

The purpose of our talk is to present a category containing "classical" (fuzzy) morphological spaces realized in the spirit of the article [1]. This provides us the flexibility to deal with transformations of fuzzy morphological spaces, which in turn can enrich the use in practical applications. As the basis for this category, we take additive groups (X, S) and (Y, T) with a selected L-fuzzy subsets $S \in L^X$ and $T \in L^Y$ playing the role of structural elements as objects and whose morphisms are forward powerset operators $f^{\rightarrow} L^X \rightarrow L^Y$ induced by homomorphisms $f : (X, S) \rightarrow (Y, T)$ such that $f(S) \leq T$. To completion, we also observe the behaviour of derived operators – fuzzy morphological opening $\mathcal{O}$ and closing $\mathcal{C}$.

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Acta Societatis Mathematicae Latviensis
Abstracts of the 13$^{\text{th}}$ Latvian Mathematical Conference, January 24-25, 2025, Riga, Latvia

WHEN A PUPIL BECOMES AN ENGINEERING STUDENT

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The article examines problems in teaching mathematics at Riga Technical University, related to changes in secondary education. From 2020, the form and content of secondary education in Latvia has changed, moving to 3 levels of education: education of general, optimal and advanced curriculum levels. The article discusses the difference in the content of mathematics programs at these levels and the results achieved by students. The statistical data of the students enrolled in two faculties has been analyzed on the level of elementary mathematics knowledge of these students, as well as the obtained results in higher mathematics in both semesters of the 1st year. The abstract highlights the problem related to the fact that students with different levels of knowledge in mathematics acquired in secondary school are taught together: in a lecture, students who studied mathematics at an advanced level in secondary school and have already mastered the basics of Higher Mathematics and students who learned mathematics at school at the optimal or even general level are in the same level audience and have not learned anything from the subject of Higher Mathematics. The article offers a possible solution to this problem by dividing students into groups according to their mathematical knowledge.

Acta Societatis Mathematicae Latviensis
Abstracts of the 13th Latvian Mathematical Conference, January 24-25, 2025, Riga, Latvia

50 YEARS OF THE OPEN MATHEMATICS OLYMPIAD

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In Latvia, the Mathematics Olympiad system has a long and rich in tradition history. Every academic year A. Liepa's Correspondence Mathematics School [1] organizes mathematics competitions and Olympiads for students from Grade 4 to 12. In school year 2024/2025 the 50th Open Mathematics Olympiad was held.

The Open Mathematics Olympiad is one of the largest school subject Olympiads in Latvia in terms of the number of students. In this Olympiad students from grade 5 to grade 12 can participate (every year, several younger grade students also participate solving 5th-grade problems; other Olympiads, see catalog of Olympiads [2], mainly are intended for students from older grades). It should be emphasized that any student who wants, can participate in Open Mathematics Olympiad, regardless of success in the stages of the State Mathematics Olympiad or in mathematics lessons. There are five problems for each grade students, they have five hours to solve them.

In recent years, the number of participants in Open Mathematics Olympiad has been growing. In the school year 20024/2025, more than 5000 students participated in the Open Mathematics Olympiad. We will discuss what changes have taken place in this Olympiad in recent years, as well as the students' results in 50th Open Mathematics Olympiad will be analyzed.

Changes in mathematics Olympiads in Latvia in recent years have also been discussed at other conferences, for example, [3], [4], [5], and more see in [6]. More about history of mathematics Olympiads in Latvia (including the Open Mathematical Olympiad) see in [7].

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